Problem 1: Solve each of the following IVPs

a) $y' = xe^{x^2-\ln(y^2)}$ $\quad y(1) = 2$
b) $3xy' + y = 12x$ $\quad y(1) = 1$
c) $y' = 1 + x + y + xy$ $\quad y(2) = 3$
d) $(e^{2y} - y)\cos x \frac{dy}{dx} = e^y \sin 2x$ $\quad y(0) = 0$
e) $y' = \frac{3x^2-y^2}{2xy}$ $\quad y(1) = 1$
f) $y' = \ln(x + \sqrt{x^2 - 1})^y$ $\quad y(1) = e$
g) $\cos^2(x)y' + y = 1$ $\quad y(0) = -3$
h) $x\frac{dy}{dx} - y = e^{x^2}y^5$ $\quad y(2) = 4$
i) $\frac{dy}{dx} = \frac{y^2\ln\left(\frac{y}{x}\right)+x^2}{xy\ln\left(\frac{y}{x}\right)}$ $\quad y(e) = 2$
j) $\frac{dy}{dx} = \frac{y^3+2xy^2+x^2y+x^3}{x(x+y)^2}$ $\quad y(2) = 5$
k) $\frac{dy}{dx} = \frac{1}{2}\frac{(x+y-1)^2}{(x+2)^2}$ $\quad y(1) = 2$
l) $(2e^{2x} + e^y)dx + (3e^{2x} + 4xe^y)dy = 0$ $\quad y(1) = 1$

Problem 2

Consider the ODE,

$$y' = \frac{y}{x} - 2y^2,$$

defined on the region $x > 0$. Suppose that at $x_1 > 0$, a solution $y(x)$is between the two curves $y = \frac{1}{x}$ and $y = 0$. Show that for all $x > x_1$, the solution is trapped between these curves.

Extra Credit Let $x_1 > 0$ and $y_1 > 0$. Prove that the solution of the IVP,

$$\begin{cases} y' = \frac{y}{x} - 2y^2, \\ y(x_1) = y_1 \end{cases}$$

is bounded as $t \to \infty$.

Problem 3

Show that $I = \frac{1}{x^2+y^2}$ is an integrating factor for

$$\big(y + xf(x^2+y^2)\big)dx + (yf(x^2+y^2) - x)dy = 0$$

Problem 4

Show that multiplying both sides of the differential equation

$$\big(P(x)y - Q(x)\big)dx + dy = 0$$

by an integrating factor $I = e^{\int P(x)dx}$ will transform the above equation into an exact equation.